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Maximum-*J* capability in a quasiaxisymmetric stellarator

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Maximum-*J* (*J* is the second adiabatic invariant) capability, i.e., the radial derivative of *J* has the same sign as that of pressure, is investigated in a quasiaxisymmetric (QA) stellarator to investigate improved confinement. Due to the existence of nonaxisymmetry of the magnetic field strength, a local maximum of *J* is created to cause the drift reversal. External controllability of the maximum-*J* condition is also demonstrated, by which the impact of magnetic configuration on turbulent transport can be studied.

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Improved plasma confinement has been realized in toroidal plasmas with the transport barrier formation [1–5]. The sharp gradient of plasma parameters is realized at the location of barrier, which is associated with the suppression of turbulent transport. The transport barrier formation in toroidal plasmas has been one of the most interesting and attractive subjects not only in plasma physics, but also in modern physics, i.e., the structural formation [6].

The turbulent fluctuation suppression in these improved modes has been considered to be consistent with theoretical prediction for the stabilization of microinstabilities. Several kinds of microinstabilities appear when directions of the diamagnetic drift and ∇B drift (B is the magnetic field strength) are in the same direction for trapped particles [7,8]. The velocity of the toroidal precession, v_ζ, can be expressed in terms of the radial derivative of the second adiabatic invariant *J*. The *J* is an invariant for a periodic bounce motion of a particle which is trapped in a magnetic mirror [9]. The stability condition for them is derived with scalar plasma pressure P as

$$\nabla P \cdot \nabla J > 0,$$

which is frequently called the maximum-*J* condition. This indicates that microinstabilities can be stabilized if the direction of the toroidal precession of trapped particles is in a favorable ($dJ/dr<0$) direction. This condition can be realized by q (safety factor) profile control (such as reversed shear tokamaks) [8], plasma cross section control (such as ellipticity) [10], plasma diamagnetism [11], and strong inhomogeneity of the radial electric field [12]. The experimental demonstration of significant increase of confinement time in a spherator [13] when trapped particles are localized in good curvature region is also considered as the remarkable example. The impacts of geometric parameters of magnetic configuration on microinstabilities and turbulent transport have been investigated both in theoretical and experimental ways for axisymmetric configurations.

Quasiaxisymmetric (QA) stellarator configurations have been widely studied recently [14–17]. In this Rapid Communication, maximum-*J* (or $dJ/dr<0$) capability of a QA configuration is examined. The magnetic configurations in stellarators are mainly provided by the external coils rather than the plasma current. This gives a unique flexibility of configuration control which has not been realized in axisymmetric configurations. This feature is favorable to investigate the impact of magnetic configuration on turbulent transport in a controlled manner. It is noted that the importance of maximum-*J* condition for the design of helical systems (including stellarators) has already been pointed out about one decade ago [18]. Its quantitative evaluation is now available to a design of a QA stellarator.

This paper is organized as follows. First, the calculation method of *J* is simply explained. An example QA configuration (vacuum case) is evaluated from the viewpoint of maximum-*J* capability. External controllability of the maximum-*J* condition through the configuration control is also examined. Finally, summary is given.

The second adiabatic invariant *J* for trapped particles is defined as

$$J \equiv \oint v_\parallel dl,$$

where dl denotes the line element of a magnetic field line and the integral is performed over a bounce period. The calculation of *J* is performed by following guiding center of low energy trapped particles whose deviation from a magnetic field line is negligibly small. The guiding center equations [19] are expressed by use of the Boozer coordinates (ψ,θ,ζ) [20], with ψ being the normalized toroidal flux and $\theta(\zeta)$ the poloidal (toroidal) angle, respectively. The motion of the guiding center is defined by five variables [(ψ,θ,ζ) for the real space, $v_\parallel$ and particle energy, W]. Since the direction (sign) of the toroidal precession is the key for the stability condition, the W dependence is not important here so that particles with fixed W are considered. Also, the integral is performed along the particle trajectory so that one out of θ and ζ dependence is omitted when the launching points of particles are specified. To obtain the radial profile of *J*, tracer particles which are to be reflected at the same B are launched from $\theta=0$. This is the location where the toroidicity-induced magnetic ripple gives the deepest well (the minimum of B on a flux surface). The launching points are distributed in the (ψ,ζ) plane, and the (ψ,ζ) dependence of *J* is analyzed (the ζ dependence must be kept due to the nonaxisymmetry of B). The particle energy is $W=10$ eV for configurations with $B_0=1$ T and $R_0=2$ m. Here B_0 is the magnetic field

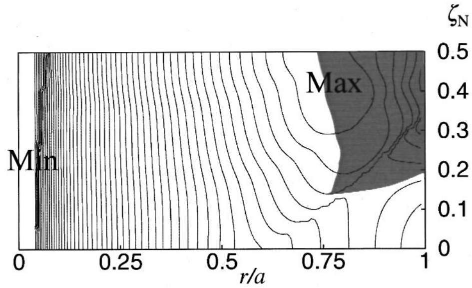

FIG. 1. Contours of J on the $(r/a,\zeta_N)$ plane for $B_{ref}=1.0$ for a QA configuration (vacuum). The tracer particles are launched from $\theta_N=0$. The region of (local) minimum and maximum are indicated with ''Min'' and ''Max,'' and the region with favorable $dJ/dr<0$ is hatched.

strength on the magnetic axis and R_0 the geometrical major radius of a device. The initial parallel velocity, $v_{\parallel,st}$, is defined as

$$v_{\parallel,st}=\sqrt{2\frac{W}{m}\left(1-\frac{B_{st}}{B_{ref}}\right)},$$

where $B_{st}(B_{ref})$ is the magnetic field strength (normalized with B_0) at the initial (bounce) point and m the particle mass.

An example QA configuration with the toroidal field period number (M) of 2 is considered in this paper after Ref. [16]. The aspect ratio of this particular configuration is about 4.3. The rotational transform lies between 0.37 and 0.4 with the slight increase towards the edge. The several nonaxisymmetric components of B appear near the edge with a few percent of the uniform magnetic field. Figure 1 shows contours of J on the $(r/a,\zeta_N)$ plane (only half of the toroidal period due to the symmetry with respect to $\zeta_N=0.5$) for the $B_{ref}=1.0$ case. The r/a is the label of the plasma radius which is normalized by the minor radius of a plasma (a) and ζ_N the toroidal angle normalized with the angle of one period $(2\pi/M)$. It shows the significantly unique feature compared to axisymmetric tokamak cases. That is, the existence of local maximum of J, which gives favorable $dJ/dr<0$ in the outer radius. This is created by the decrease of J in the edge region. In the following, this unique feature is considered based on magnetic field structure and tracer particle trajectories.

Figure 2 shows contours of the measure of the toroidal inhomogeneity of B, $\gamma\equiv(1/B)(\partial B/\partial\zeta)$, on magnetic surfaces $(-0.25\leqslant\theta_N\leqslant0.25$ for 2 toroidal periods) at two radii. The trajectories (bold straight lines) of particles launched from $\zeta_N=0$ and 0.5 on each surface are also shown. The contours of $\gamma=0$ are shown by dashed curves and interval of contours is 2.5%. The region with $\gamma>0(<0)$ indicates that B increases (decreases) as ζ_N is increased. This contour is valuable in order to clarify the contribution from nonaxisymmetry of B to $v_\parallel$ through the conservation of magnetic moment. The magnetic moment is an adiabatic invariant associated with the circular current due to the cyclotron motion of a gyrating particle. It is seen that the toroidal inhomogeneity of B is enhanced as r/a is increased (compare the hatched region with $\gamma>5\%$). The nonaxisymmetry of B

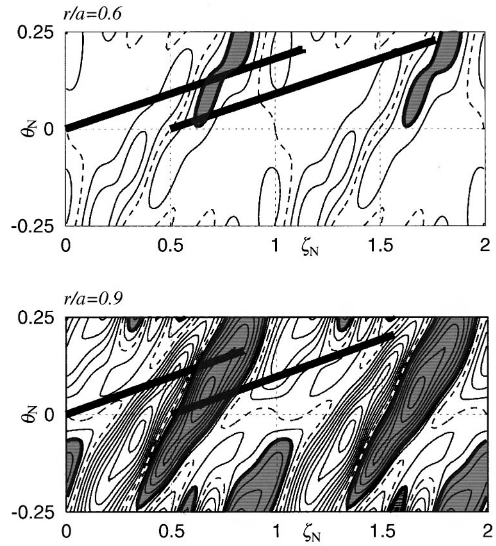

FIG. 2. Contours of $\gamma\equiv(1/B)(\partial B/\partial\zeta)$ on the (ζ_N,θ_N) plane $(-0.25\leqslant\theta_N\leqslant0.25$ for two toroidal periods) at $r/a=0.6$ and 0.9. The outer side of a torus corresponds to $\theta_N=0$. The contours of $\gamma=0$ are shown by dashed curves and the interval of contours is 2.5%. The region with $\gamma>5\%$ is hatched. Trajectories of tracer particles launched from $\zeta_N=0$ and 0.5 are also shown by thick solid lines.

gives rise to a localized region with enhanced γ. The particle which is launched from $\zeta_N=0.5$ passes this region with enhanced $\gamma>0$ in their initial and final phase of motion. This is more pronounced for outer radius and J becomes smaller compared to that of inner radius by reducing $v_\parallel$ largely. This feature is unique in QA stellarators, which can be utilized as the new way to establish favorable $dJ/dr<0$ region (drift reversal). It is noted that the nonaxisymmetric contribution is less influential for particles which are launched from $\zeta_N=0$ because they go through a region with smaller γ (cf. Fig. 2). This gives monotonic increase of J towards the edge for $\zeta_N\sim0$.

It is important to have the controllability of maximum-J so that externally controlled experiments are possible. A proposed QA stellarator device has several kinds of coils such as main modular coils, auxiliary coils, and vertical field coils to control properties of magnetic configurations (see Ref. [16] for details). Here, the controllability of maximum-J region is examined by taking an inward-shifted configuration as an example. Figure 3 shows contours of J on the $(r/a,\zeta_N)$ plane

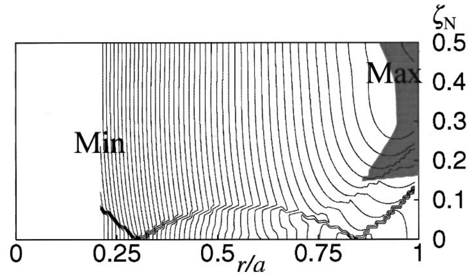

FIG. 3. Contours of J on the $(r/a,\zeta_N)$ plane for $B_{ref}=1.0$ for an inward-shifted QA configuration (vacuum).

for the $B_{ref}=1.0$ case (vacuum case). The local maximum of J shifts towards the edge and the maximum-J region (hatched) is radially shrunk as compared to that shown in Fig. 1. This is because the nonaxisymmetry of B is weakend in an inward-shifted configuration. Thus, external controllability of the maximum-J region through magnetic configuration control is demonstrated. This enhances attractiveness and significance of experiments of a QA stellarator to investigate improved confinement.

The maximum-J capability has been investigated in a quasiaxisymmetric (QA) stellarator configuration to investigate improved confinement through the possibility of turbulent transport suppression realized by the drift reversal. The local maximum of J is created to give favorable $dJ/dr<0$, which is due to the nonaxisymmetry of B. This is a different way to realize the drift reversal, which is not the case in axisymmetric configurations. External controllability of maximum-J region through magnetic configuration control is also demonstrated. This allows us the systematic study for the impact of magnetic configuration on turbulent transport.

Finally, it is noted that the results in this paper have been obtained in an example QA configuration. A similar maximum-J property has been found in a few other examples [21,22]. It is a very interesting subject to examine other QA configurations to investigate whether the maximum-J capability holds in general in QA configurations. The maximum-J capability of present stellarators or heliotrons should also be the interesting subject to consider the possibility of drift reversal in a wide range of helical configurations.

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